Van Hiele Geometric Thinking Levels among Senior High School Students in Ghana: Evidence from the Asante Akim South Municipality

Charles Siaw Obeng
Department of Mathematics Education
Kwame Nkrumah University of Science and Technology (KNUST), Kumasi, Ghana
ORCID: https://orcid.org/0009-0003-7313-3250.

Abstract
This study examined the Van Hiele geometric thinking levels attained by Year Three Senior High School students in the Asante Akim South Municipality of Ghana. **A** quantitative descriptive cross-sectional design was adopted. A total of 320 students**,** selected through stratified random sampling**,** completed the Van Hiele Geometry Test (VHGT) developed by Usiskin (1982)**.** Descriptive statistics were used to analyse the data. The findings revealed that 110 students (34.38**%)** attained Level 1 (Visualization)**,** 140 students (43.75%) attained Level 2 (Analysis**)**, and 70 students **(**21.88%) attained Level 3 (Informal Deduction)**.** The study adopted **Usiskin's (**1982) numbering convention**,** in which the first Van Hiele level is designated as Level 1 rather than Level 0**.** No student attained Level 4 (Deduction); consequently, Level 5 (Rigor) was also **not** attained**.** The mean Van Hiele geometric thinking level was 2.13 (*SD* = 0.82)**,** indicating that the majority of students operated at the Analysis level. The findings underscore the need for instructional approaches that promote students' progression to higher levels of geometric reasoning.
Keywords: Van Hiele levels; geometric thinking; Van Hiele Geometry Test; senior high school students; Ghana; quantitative descriptive study.

Introduction

Background of the study

Geometry is a fundamental branch of mathematics that develops learners' ability to reason about shapes, space, and spatial relationships. Geometry is evident in both natural and human-made structures and is essential in science, technology, and daily life (Ablonski & Ludwig, 2023). Despite its importance, students' performance in geometry, particularly their level of geometric thinking, remains a major concern in many countries, including Ghana. The Van Hiele theory, developed by Van Hiele (1986), describes the hierarchy of geometric thinking in five Levels; Level 0 (Pre-visualization), Level 1 (Visualization), Level 2 (Analysis), Level 3 (Informal Deduction) and Level 4 (Deduction). Students are expected to progress through these levels with appropriate instruction and experiences**.**

Statement of the problem

Although several studies have examined students' geometric thinking in Ghana, most have focused on relationships between variables such as gender, school or age. Limited evidence exists on the actual levels for Van Hiele geometric thinking attained by students in the Asante Akim South Municipality. Without such baseline information, it is difficult for teachers and curriculum planners to design context-appropriate instructional strategies to promote the development of higher geometric thinking levels.

Objectives of the study

To describe the Van Hiele geometric thinking levels of Year Three Senior High School students in the Asante Akim South Municipality.

Research questions

What Van Hiele geometric thinking levels are attained by Year Three Senior High School students in the Asante Akim South Municipality?

Literature Review

The Van Hiele Theory of Geometric Thinking

The Van Hiele model (Van Hiele, 1986) describes five hierarchical levels of geometric thinking; Level 0 (Pre-visualization), Level 1 (Visualization), Level 2 (Analysis), Level 3 (Informal Deduction) and Level 4 (Deduction). Progression from one level to another occurs through appropriate instruction and learning experiences.

Constructivist Learning Theory

Constructivism posits that learners construct knowledge through active engagement with tasks and social interaction (Vygotsky, 1978). Meaningful geometric thinking develops when students interact with tasks that are slightly above their current level of understanding.

Piaget's Cognitive Development Theory

Piaget (1970) argued that cognitive development occurs in stages. Formal operational thinking essential for higher geometric reasoning typically develops during adolescence.

Empirical Studies on Students' Geometric Thinking Levels

International studies (e.g., Usiskin, 1987) consistently report that most students are at the visualization or analysis levels, with very few reaching deductive reasoning.

Study in Ghana

Asemani et al. (2017) found that a large proportion of Ghanaian Senior High School students had not attained even the lowest Van Hiele level, while only a small percentage reached higher levels.

Conceptual Framework of the Study

The conceptual framework for this study is based on the Van Hiele theory of geometric thinking (Van Hiele, 1986), which explains that students progress through a hierarchy of geometric thinking levels as a result of appropriate instruction and meaningful learning experiences. The theory proposes that learners develop from recognizing geometric figures based on their appearance to analysing their properties and eventually engaging in logical reasoning and formal deduction.

In this study, geometry learning experiences, including classroom instruction, learning activities, and the use of instructional resources, serve as the instructional inputs that facilitate students' development of geometric thinking. These learning experiences influence students' progression through the Van Hiele levels. The outcome of this process is reflected in the students' attained Van Hiele geometric thinking levels**,** which were measured using the Van Hiele Geometry Test (VHGT). The students' geometric thinking levels were described using frequencies, percentages, the mean, and the standard deviation. The framework therefore illustrates that appropriate geometry instruction and learning experiences support students' progression through the hierarchical levels of geometric thinking, resulting in different distributions of Van Hiele levels among Year Three Senior High School students.

Interpretation within the Study

The conceptual framework guided the study by illustrating how geometry instruction and learning experiences influence students' geometric thinking. It provided the basis for describing the distribution of students across the Van Hiele levels using descriptive statistics. The framework therefore supports the interpretation of students' attained levels of geometric thinking and highlights the importance of effective instructional experiences in promoting higher levels of geometric reasoning.

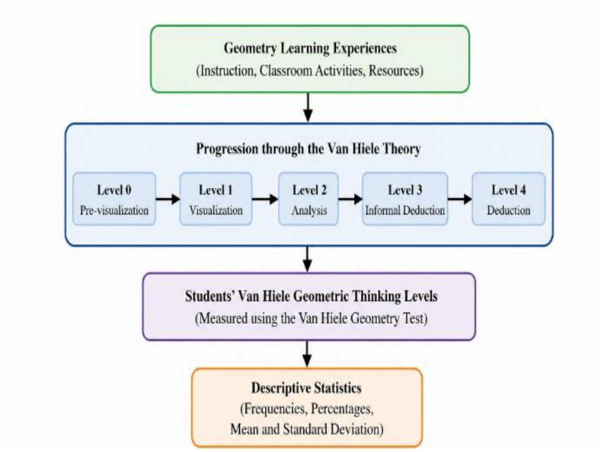

Figure 1. Conceptual framework illustrating the relationship between geometry learning experiences and students' Van Hiele geometric thinking levels.

Research Gap

While studies have described levels in other contexts or examined relationships with other variables, there is limited descriptive evidence on the actual distribution of Van Hiele levels among Year Three students in the Asante Akim South Municipality.

Research Methodology

Research design

A quantitative descriptive cross-sectional design was employed to assess the geometric thinking levels of the students using the Van Hiele Geometry Test (VHGT).

Population

The study was conducted in the Asante Akim South Municipality of the Ashanti Region, Ghana. The target population comprised all Year Three (SHS 3) students enrolled in the three public senior high schools in the municipality during the 2024/2025 academic year. Public senior high schools were selected because they implement the standardized curriculum prescribed by the Ghana Education Service (GES), thereby providing a comparable educational context for investigating students' geometric thinking levels.

The total population of Year Three students across the three schools was N = 898. School A had N_1 = 515 students, School B had N_2 = 298 students, and School C had N_3 = 85 students.

Sampling Procedure and Sample

A combination of purposive and stratified random sampling techniques was employed.

First, the three public senior high schools were purposively selected because they represented the public second-cycle institutions within the municipality. School A is located in the municipal capital, whereas Schools B and C are located in rural communities, thereby providing representation of different geographical contexts. Second, proportionate stratified random sampling was used to select participants from each school. The student population within each school was stratified by gender, after which simple random sampling was used to select participants proportionately from each stratum (Cohen et al., 2007).

The proportional allocation of the sample was based on the size of the Year Three student population in each school using

$$n_h = \frac{N_h}{N} \times n$$

where n_h is the number of students selected from a particular school, N_h is the Year Three student population of that school, N is the total Year Three population across the three schools, n is the total sample size (320) and h is the given stratum.

Then,

$$n_1 = \frac{515}{898} \times 320 = 184$$

Using this procedure, 184 students (35.73%) were selected from School A, 106 students (35.57%) from School B, and 30 students (35.29%) from School C, giving a total sample of 320 students. The final sample comprised 186 females (20.71%) and 134 males (14.92%).

Table 1

Population and Proportional Allocation of the Sample

Stratum	School	Population (N)	Sample (n)	Sampling Fraction (%)
h = 1	School A	515	184	35.73
h = 2	School B	298	106	35.57
h = 3	School C	85	30	35.29
Total		**898**	**320**	**35.63**

Reliability and Validity of the Instrument

The Van Hiele Geometry Test (VHGT) developed by Usiskin (1982**)** was used to assess students' geometric thinking levels. A pilot study involving 37 Year Three (SHS 3) students was conducted to establish the reliability of the instrument. Internal consistency was assessed using the Kuder–Richardson Formula 20 (KR-20**)**, yielding a reliability coefficient of 0.77**,** which indicates acceptable reliability for research purposes.

The face and content validity of the instrument were established through expert review by **two** experienced mathematics educators**.** The experts evaluated the relevance, clarity, and appropriateness of the test items in relation to the Senior High School Mathematics Curriculum in Ghana. Their recommendations were incorporated before the instrument was administered.

Ethical Considerations

Ethical approval to conduct the study was obtained from the relevant school authorities before data collection. Informed consent was obtained from all participants after the purpose of the study had been explained. Participation was voluntary, and participants were assured of confidentiality and anonymity. Students were informed of their right to withdraw from the study at any stage without penalty. Upon completion of the study, a summary of the findings was shared with the participating schools.

Instrument

The Van Hiele Geometry Test (VHGT) developed by Usiskin (1982**)** was used to assess students' geometric thinking levels. The instrument consists of 25 multiple-choice items, with **five items** measuring each of the five hierarchical Van Hiele levels of geometric thinking. Students' responses were scored according to Usiskin's classification procedure, whereby a student was considered to have attained a particular Van Hiele level after correctly answering at least three of the five items associated with that level. Each student was then assigned the highest Van Hiele level attained.

The present study adopted Usiskin's (1982) numbering convention, in which the five levels are designated as Level 1 (Visualization), Level 2 (Analysis), Level 3 (Informal Deduction), Level 4 **(**Deduction), and Level 5 (Rigor). Consequently, Level 0 does not appear in this study because **it** is not part of Usiskin's numbering system. The reference to Level 0 is associated with the **original** Van Hiele model, where the visualization stage is labelled Level 0. In contrast, Usiskin renumbered the levels to begin at Level 1, and this convention has been widely adopted in studies using the VHGT. Although the VHGT assesses all five levels, the analysis showed that the participants attained only Levels 1 (Visualization), 2 (Analysis), and 3 (Informal Deduction). No student attained Level 4 **(**Deduction), and consequently Level 5 (Rigor) was not attained because the Van Hiele model is hierarchical. Therefore, only Levels 1–3 are reported in the results.

Data Collection Procedure

Permission to conduct the study was obtained from the heads of the selected senior high schools in the Asante Akim South Municipality before data collection commenced. The Van Hiele Geometry Test (VHGT) developed by Usiskin (1982) was administered to the participants in February 2024 under standardized examination conditions. Students were given 45 minutes to complete the 25-item multiple-choice test.

The VHGT consists of 25 items**,** with five items representing each Van Hiele geometric thinking level. Students' responses were scored according to the Van Hiele scoring procedure proposed by Usiskin (1982). A student was considered to have attained a particular Van Hiele level if he or she answered at least three of the five items correctly for that level. Based on this criterion, each student was assigned the highest Van Hiele geometric thinking level attained, thereby producing one geometric thinking level for each of the 320 students in the study.

The individual Van Hiele level attained by each student constituted the dataset entered into IBM SPSS Statistics version 26. Consequently, the descriptive statistics, including the mean ($M = 2.13$) and standard deviation **(**$SD = 0.82$)**,** were computed from the 320 individual student observations**,** where each student represented one case in the dataset. The frequencies and percentages reported in the results section were subsequently generated by summarizing the number of students who attained each Van Hiele level.

For example, if four students attained Level 2 and five students attained Level 1, SPSS first recorded each student as an individual observation (e.g., 2, 2, 2, 2, 1, 1, 1, 1, 1). The software then

calculated the overall descriptive statistics from all student observations before producing the frequency distribution across the Van Hiele levels.
Therefore, the reported mean (2.13) and standard deviation (0.82) summarize the individual student-level data, whereas the frequency table is a descriptive summary showing the number of students who attained each Van Hiele level. The frequency table itself was not the dataset used to compute the SPSS descriptive statistics.

Results

Table 2

Distribution of Year Three Senior High School Students Across Van Hiele Geometric Thinking Levels (N = 320)

Van Hiele Level	Description	*N*	%
Level 1	Visualization	110	34.38
Level 2	Analysis	140	43.75
Level 3	Informal Deduction	70	21.88
Total		**320**	**100.00**

Table 3

Descriptive Statistics

Statistic	Value
Mean	2.13
Standard deviation	0.82

Results

Table 4 presents the distribution of Year Three Senior High School students across the Van Hiele geometric thinking levels. Of the 320 students, 110 (34.38%) attained Level 1 (Visualization), 140 (43.75%) attained Level 2 (Analysis), and 70 (21.88%) attained Level 3 (Informal Deduction). The results indicate that Level 2 (Analysis) was the most frequently attained level, representing nearly half of the participants.
The study adopted Usiskin's (1982) numbering convention for the Van Hiele model, in which the first level is designated as Level 1 rather than Level 0. Consequently, Level 0 was not included in the analysis. Although the VHGT measures five hierarchical Van Hiele levels, no student attained Level 4 (Deduction). Because progression through the Van Hiele hierarchy is sequential, no student attained Level 5 (Rigor). Therefore, only Levels 1–3 are reported in Table 4**.**
The mean Van Hiele geometric thinking level was **2**.13 (SD = 0.82), indicating that, on average, students operated at the Analysis level.

Discussion of Findings

The findings indicate that the Analysis level (Level 2) represents the dominant stage of geometric thinking among the students in this study. The descriptive statistics further support this conclusion, with a mean Van Hiele level of 2.13 (SD = 0.82), indicating that the average student had attained the Analysis level. At this level, students are able to identify and describe the properties of geometric figures but often experience difficulty in constructing logical arguments and formal geometric proofs. Although 21.88% of the students attained the Informal Deduction level (Level

3), the relatively small proportion suggests that many learners have not yet developed the higher-order reasoning skills required for advanced geometric problem-solving. This interpretation is supported by the mean Van Hiele level of 2.13 (SD = 0.82), indicating that the average student operated at the Analysis level rather than at the higher levels of geometric reasoning.

The observed distribution of Van Hiele levels highlights the need for instructional approaches that deliberately support students' progression through the hierarchical levels of geometric thinking. According to Van Hiele (1986), advancement from one level to the next depends primarily on carefully sequenced instruction and meaningful learning experiences rather than simply on increased exposure to geometry. Consequently, geometry teaching should provide students with opportunities to explore geometric relationships, justify their reasoning, and engage in activities that promote logical thinking.

Comparison with Literature

The findings of this study are consistent with those of Adu et al. (2020) and the theoretical framework proposed by Van Hiele (1986), both of which indicate that learners typically progress gradually through the hierarchical levels of geometric thinking, with many secondary school students remaining at the Analysis level. Similarly, Burger and Shaughnessy (1986) reported that a substantial proportion of secondary school students operate at the Analysis and Informal **Deduction** levels, with relatively few attaining higher levels of geometric reasoning. The present findings therefore reinforce existing evidence that many students have not yet developed advanced deductive reasoning in geometry and highlight the importance of instructional practices that facilitate progression to higher Van Hiele levels.

Contribution to Knowledge

This study contributes to mathematics education by expanding the geographical coverage of research on Van Hiele geometric thinking through its focus on a semi-urban municipality in Ghana. It also provides post-COVID baseline evidence on the distribution of Van Hiele geometric thinking levels among Year Three Senior High School students. The findings extend the limited body of research on geometric thinking in Ghana and provide evidence that can inform geometry instruction, curriculum planning, and future research aimed at improving students' geometric reasoning.

Conclusions

Based on the findings of this study, it is concluded that the majority of Year Three Senior High School students in the Asante Akim South Municipality operate at the Analysis level (Level 2) of the Van Hiele geometric thinking model. Although some students have progressed to the Informal Deduction level (Level 3), relatively few have attained higher levels of geometric reasoning. These findings suggest that many students have developed the ability to identify and analyse the properties of geometric figures but require further instructional support to develop higher-order reasoning and deductive thinking. The study highlights the need for geometry instruction that deliberately promotes students' progression through the hierarchical levels of the Van Hiele model**.**

Recommendations

Based on the findings of this study, the following recommendations are made.

First, mathematics educators and curriculum planners should design and implement geometry instruction that is explicitly aligned with the Van Hiele theory. Instruction should be sequenced to support students' progression through the hierarchical levels of geometric thinking, particularly from the Analysis level to the Informal Deduction level.

Second, teachers should incorporate instructional activities that promote spatial reasoning, visualization, exploration, and logical reasoning. Learner-centred approaches, including the use of manipulatives, dynamic geometry software, and collaborative problem-solving activities, should be employed to enhance students' understanding of geometric concepts.

Third, school administrators should organize regular professional development programmes and workshops to strengthen teachers' knowledge of the Van Hiele theory and effective instructional strategies for promoting higher levels of geometric thinking. Finally, future studies should examine instructional interventions that facilitate students' progression from the Analysis level to the Informal Deduction and Deduction levels.

References

Ablonski, A. M., & Lockwig, M. (2023). *Teaching and learning geometry in school mathematics*. *Journal of Mathematics Education, 16*(2), 145–163.

Adu, E., Mensah, J. K., & Boateng, P. A. (2020). *Geometric thinking levels among senior high school students in Ghana*. *African Journal of Educational Studies*, *15*(2), 45–60.

Asemani, A., Oppong, R. A., & Asiedu-Addo, S. K. (2017). *An analysis of SHS students' geometric thinking levels based on the Van Hiele model*. *Ghana Journal of Mathematics Education*, *3*(2), 55–66.

Burger, W. F., & Shaughnessy, J. M. (1986). *Characterizing the Van Hiele levels of development in geometry*. *Journal for Research in Mathematics Education, 17*(1), 31–48.

Piaget, J. (1970). *Science of education and the psychology of the child*. Orion Press.

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry* (Final report of the Cognitive Development and Achievement in Secondary School Geometry Project). University of Chicago.

Van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. Academic Press.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.

APPENDICES

Appendix A:

Section A:

Adapted Van Hiele Geometry Test (Based on Usiskin, 1982)

This instrument was adapted from Usiskin (1982) to assess students' levels of geometric thinking. It consists of 25 multiple-choice items distributed across the five Van Hiele levels: Visualization, Analysis, Informal Deduction, Deduction, and Rigor. Students select the one best answer for each item.

Level 1 – Visualization

1. Which of the following figures is a square?
   (a) A shape with all sides equal and all right angles
   (b) A shape with two long sides and two short sides
   (c) A triangle with equal sides
   (d) A shape with curved sides

2. Which figure below is not a triangle?
   (a) A shape with three sides
   (b) A figure with four corners
   (c) A figure with three corners
   (d) A closed figure made of three straight lines

3. A rectangle can best be recognized because:
   (a) It looks like a box
   (b) It has four right angles
   (c) It has all sides equal
   (d) It has only two right angles

4. Which figure is a circle?
   (a) A figure with three equal sides
   (b) A figure with no corners
   (c) A figure with four sides
   (d) A figure with two equal sides

5. Which of the following figures is not a polygon?
   (a) Triangle
   (b) Rectangle
   (c) Circle
   (d) Pentagon

Level 2 – Analysis

6. Which statement about a parallelogram is true?
   (a) It has one pair of parallel sides
   (b) Opposite sides are parallel and equal
   (c) All angles are right angles
   (d) All sides are unequal

7. The diagonals of a rectangle:
   (a) Are equal in length
   (b) Are always perpendicular
   (c) Never meet
   (d) Are longer on one side

8. In a rhombus, the diagonals:
   (a) Are equal and perpendicular
   (b) Are not equal but perpendicular
   (c) Are not equal and not perpendicular
   (d) Are equal but not perpendicular

9. The sum of interior angles of a triangle is:
   (a) 90°
   (b) 180°
   (c) 270°
   (d) 360°

10. A trapezium (trapezoid) has:
   (a) All sides equal
   (b) Two pairs of parallel sides
   (c) One pair of parallel sides
   (d) No parallel sides

Level 3 – Informal Deduction

11. Every square is a rectangle because:
   (a) Both have four equal sides
   (b) Both have all angles equal to 90°
   (c) A square has all properties of a rectangle
   (d) They look the same

12. Which statement is correct?
   (a) Every rectangle is a rhombus
   (b) Every square is a rhombus
   (c) Every rhombus is a rectangle
   (d) Every parallelogram is a rectangle

13. If a figure is a rectangle, then:
   (a) Its opposite sides are equal and parallel
   (b) It has only one right angle
   (c) Its diagonals never meet
   (d) Its diagonals are unequal

14. Which statement correctly relates triangles?
   (a) All equilateral triangles are isosceles
   (b) All isosceles triangles are equilateral
   (c) Some scalene triangles are equilateral
   (d) Right triangles are always equilateral

15. A kite is different from a parallelogram because:
   (a) It has opposite sides parallel
   (b) It has two pairs of adjacent equal sides

(c) It has all sides equal
(d) It has all right angles

Level 4 – Deduction

16. If two triangles have all three sides equal, then they are:
(a) Congruent
(b) Similar only
(c) Different in size
(d) Right-angled

17. The sum of angles in a quadrilateral equal:
(a) 90°
(b) 180°
(c) 270°
(d) 360°

18. If two lines are parallel and cut by a transversal, alternate interior angles are:
(a) Equal
(b) Complementary
(c) Supplementary
(d) Unequal

19. The diagonals of a square are:
(a) Equal and perpendicular
(b) Unequal and perpendicular
(c) Equal and not perpendicular
(d) Unequal and not perpendicular

20. If the radius of a circle is doubled, its area:
(a) Doubles
(b) Becomes four times larger
(c) Remains the same
(d) Is halved

Level 5 – Rigor

21. In non-Euclidean geometry, the sum of interior angles of a triangle can be:
(a) Always 180°
(b) Less or greater than 180°
(c) Exactly 360°
(d) Cannot be measured

22. Two straight lines on a sphere (great circles):
(a) Never meet
(b) Meet at two points
(c) Are parallel everywhere
(d) Are perpendicular

23. In Euclidean geometry, through a point not on a line there is:
    (a) One parallel line
    (b) No parallel line
    (c) Many parallel lines
    (d) A perpendicular line only

24. A theorem in geometry is:
    (a) A definition
    (b) A statement that can be proved
    (c) A postulate
    (d) A guess

25. If two lines are perpendicular to the same line, they are:
    (a) Parallel to each other
    (b) Perpendicular to each other
    (c) Intersecting
    (d) Skew

Appendix C:
MARKING SCHEME - Instructions to Examiner:
Use the answers to score each respondent's highest Van Hiele level (from Level 1 to Level 5).
Based on: Usiskin (1982) – Cognitive Development and Achievement in Secondary School Geometry (CDASSG)

Level	Van Hiele Level	Question Nos.	Expected Student Performance	Mark Allocation (per question)	Level Mastery Criteria
L1	**Recognition (Visualization)**	1–5	Correctly identifies shapes (e.g., triangle, square, and circle) without necessarily knowing properties.	1 mark each (Total = 5 marks)	Achieves at least 4/5 correct responses.
L2	**Analysis (Properties)**	6–10	Recognizes and states correct properties of figures (e.g., "A rectangle has opposite sides equal").	1 mark each (Total = 5 marks)	Achieves at least 3/5 correct responses.
L3	**Order / Informal Deduction (Abstraction)**	11–15	Classifies shapes correctly (e.g., "All squares are rectangles but	1 mark each (Total = 5 marks)	Achieves at least 3/5 correct responses.

			not all rectangles are squares”).		
L4	**Deduction (Formal Deduction)**	16–20	Provides valid logical reasoning or proofs (e.g., applies Pythagoras theorem or angle sum theorem).	1 mark each (Total = 5 marks)	Achieves at least 3/5 correct responses.
L5	**Rigor (Axiomatic Systems)**	21–25	Demonstrates ability to compare Euclidean and non-Euclidean systems; uses axioms or postulates correctly.	1 mark each (Total = 5 marks)	Achieves at least 3/5 correct responses.

1