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Van Hiele Geometric Thinking Levels Among Senior High School Students in Ghana: Evidence from the Asante Akim South Municipality

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Abstract

This study examined the Van Hiele geometric thinking levels attained by Year Three Senior High School students in the Asante Akim South Municipality of Ghana. A quantitative descriptive cross-sectional design was adopted. A total of 320 students were selected using purposive and proportionate stratified random sampling. Descriptive statistics were used to analyze the data. The findings revealed that 110 students (34.38%) attained Level 1 (Visualization), 140 students (43.75%) attained Level 2 (Analysis), and 70 students (21.88%) attained Level 3 (Informal De- duction). The study adopted the numbering convention in which the first Van Hiele level is designated as Level 1 rather than Level 0. No student attained Level 4 (Deduction); consequently, Level 5 (Rigor) was also not attained. The mean Van Hiele geometric thinking level was 1.875 (SD = 0.741), indicating that the largest proportion of students operated at the Analysis level. The findings underscore the need for instructional approaches that promote students' progression to higher levels of geometric reasoning.

Keywords

Van Hiele Levels, Geometric Thinking, Van Hiele Geometry Test, Senior High School Students, Ghana, Quantitative Descriptive Study

1. Introduction

1.1. Background of the Study

Geometry is a fundamental branch of mathematics that develops learners' ability to reason about shapes, space, and spatial relationships. Geometry is evident in both natural and human-made structures and is essential in science, technology,

and daily life [2]. Despite its importance, students' performance in geometry, particularly their level of geometric thinking, remains a major concern in many countries, including Ghana. The Van Hiele theory, developed by Van Hiele [3], de- scribes the hierarchy of geometric thinking in five Levels; Level 0 (Pre-visualization), Level 1 (Visualization), Level 2 (Analysis), Level 3 (Informal Deduction) and Level 4 (Deduction). Students are expected to progress through these levels with appropriate instruction and experiences.

1.2. Statement of the Problem

Although several studies have examined students' geometric thinking in Ghana, most have focused on relationships between variables such as gender, school or age. Limited evidence exists on the actual levels for Van Hiele geometric thinking attained by students in the Asante Akim South Municipality. Without such base-line information, it is difficult for teachers and curriculum planners to design con-text-appropriate instructional strategies to promote the development of higher geometric thinking levels.

1.3. Objectives of the Study

To describe the Van Hiele geometric thinking levels of Year Three Senior High School students in the Asante Akim South Municipality.

1.4. Research Questions

What Van Hiele geometric thinking levels are attained by Year Three Senior High School students in the Asante Akim South Municipality?

2. Literature Review

2.1. The Van Hiele Theory of Geometric Thinking

The Van Hiele model [3] describes five hierarchical levels of geometric thinking; Level 0 (Pre-visualization), Level 1 (Visualization), Level 2 (Analysis), Level 3 (In-formal Deduction) and Level 4 (Deduction). Progression from one level to an-other occurs through appropriate instruction and learning experiences.

2.2. Constructivist Learning Theory

Constructivism posits that learners construct knowledge through active engagement with tasks and social interaction [4]. Meaningful geometric thinking develops when students interact with tasks that are slightly above their current level of understanding.

2.3. Piaget's Cognitive Development Theory

Piaget [5] argued that cognitive development occurs in stages. Formal operational thinking essential for higher geometric reasoning typically develops during adolescence.

2.4. Empirical Studies on Students' Geometric Thinking Levels

2.4.1. International Studies

International studies (e.g., Usiskin [6]) consistently report that most students are at the visualization or analysis levels, with very few reaching deductive reasoning.

2.4.2. Study in Ghana

Asemani *et al*. [7] found that a large proportion of Ghanaian Senior High School students had not attained even the lowest Van Hiele level, while only a small percentage reached higher levels.

2.5. Conceptual Framework of the Study

The conceptual framework for this study is based on the Van Hiele theory of geometric thinking [3], which explains that students' progress through a hierarchy of geometric thinking levels as a result of appropriate instruction and meaningful learning experiences. The theory proposes that learners develop from recognizing geometric figures based on their appearance to analyzing their properties and eventually engaging in logical reasoning and formal deduction.

In this study, geometry learning experiences, including classroom instruction, learning activities, and the use of instructional resources, serve as the instructional inputs that facilitate students' development of geometric thinking. These learning experiences influence students' progression through the Van Hiele levels. The outcome of this process is reflected in the students' attained Van Hiele geometric thinking levels, which were measured using the Van Hiele Geometry Test (VHGT). The students' geometric thinking levels were described using frequencies, percentages, the mean, and the standard deviation. The framework therefore illustrates that appropriate geometry instruction and learning experiences support students' progression through the hierarchical levels of geometric thinking, resulting in different distributions of Van Hiele levels among Year Three Senior High School students.

Interpretation within the Study

The conceptual framework guided the study by illustrating how geometry instruction and learning experiences influence students' geometric thinking. It provided the basis for describing the distribution of students across the Van Hiele levels using descriptive statistics. The framework therefore supports the interpretation of students' attained levels of geometric thinking and highlights the importance of effective instructional experiences in promoting higher levels of geometric reasoning. The conceptual framework is presented in **Figure 1**.

2.6. Research Gap

While studies have described levels in other contexts or examined relationships with other variables, there is limited descriptive evidence on the actual distribution of Van Hiele levels among Year Three students in the Asante Akim South Municipality.

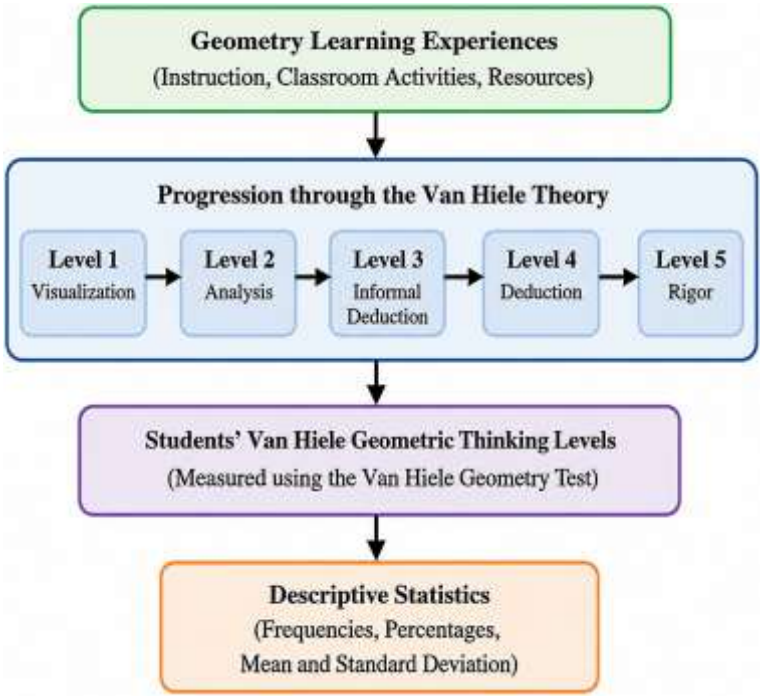


Figure 1. Conceptual framework illustrating the relationship between geometry learning experiences and students' Van Hiele geometric thinking levels.

3. Research Methodology

3.1. Research Design

A quantitative descriptive cross-sectional design was employed to assess the geometric thinking levels of the students using the Van Hiele Geometry Test (VHGT).

3.2. Population

The study was conducted in the Asante Akim South Municipality of the Ashanti Region, Ghana. The target population comprised all Year Three (SHS 3) students enrolled in the three public senior high schools in the municipality during the 2023/2024 academic year. Public senior high schools were selected because they implement the standardized curriculum prescribed by the Ghana Education Service (GES), thereby providing a comparable educational context for investigating students' geometric thinking levels.

The total population of Year Three students across the three schools was 898. School A, (515) students, School B, (298) students, and School C, (85) students.

3.3. Sampling Procedure and Sample

A combination of purposive and stratified random sampling techniques was employed.

First, the three public senior high schools were purposively selected because

they represented the public second-cycle institutions within the municipality. School A is located in the municipal capital, whereas Schools B and C are located in rural communities, thereby providing representation of different geographical contexts. Second, proportionate stratified random sampling was used to select participants from each school. The student population within each school was stratified by gender, after which simple random sampling was used to select participants proportionately from each stratum [8].

The proportional allocation of the sample was based on the size of the Year Three student population in each school. Using this procedure, 184 students (35.73%) were selected from School A, 106 students (35.57%) from School B, and 30 students (35.29%) from School C, giving a total sample of 320 students. The final sample comprised 186 females (58.13%) and 134 males (41.88%). The school-level allocation is presented in **Table 1**.

Table 1. Population and proportional allocation of the sample.

Stratum	School	Population (N)	Sample (n)	Sampling Fraction (%)
h = 1	School A	515	184	35.73
h = 2	School B	298	106	35.57
h = 3	School C	85	30	35.29
Total		898	320	35.63

3.4. Reliability and Validity of the Instrument

The Van Hiele Geometry Test (VHGT) developed by Usiskin [1] was used to assess students' geometric thinking levels. A pilot study involving 37 Year Three (SHS 3) students was conducted to establish the reliability of the instrument. Internal consistency was assessed using the Kuder-Richardson Formula 20 (KR-20), yielding a reliability coefficient of 0.77, which indicates acceptable reliability for research purposes.

The face and content validity of the instrument were established through expert review by two experienced mathematics educators**.** The experts evaluated the relevance, clarity, and appropriateness of the test items in relation to the Senior High School Mathematics Curriculum in Ghana. Their recommendations were incorporated before the instrument was administered.

3.5. Ethical Considerations

Ethical approval to conduct the study was obtained from the relevant school authorities before data collection. Informed consent was obtained from all participants after the purpose of the study had been explained. Participation was voluntary, and participants were assured of confidentiality and anonymity.

Students were informed of their right to withdraw from the study at any stage without penalty. Upon completion of the study, a summary of the findings was shared with the participating schools.

3.6. Instrument

The Van Hiele Geometry Test (VHGT) developed by Usiskin [1] was used to assess students' geometric thinking levels. The instrument consists of 25 multiple-choice items, with five items measuring each of the five hierarchical Van Hiele levels of geometric thinking. Students' responses were scored according to Usiskin's classification procedure, whereby a student was considered to have attained a particular Van Hiele level after correctly answering at least three of the five items associated with that level. Each student was then assigned the highest Van Hiele level attained. The scoring criteria used to determine the highest Van Hiele level attained by each student are presented in Table B1.

The present study adopted Usiskin's [1] numbering convention, in which the five levels are designated as Level 1 (Visualization), Level 2 (Analysis), Level 3 (Informal Deduction), Level 4 (Deduction), and Level 5 (Rigor). Consequently, Level 0 does not appear in this study because it is not part of Usiskin's numbering system. The reference to Level 0 is associated with the original Van Hiele model, where the visualization stage is labelled Level 0. In contrast, Usiskin renumbered the levels to begin at Level 1, and this convention has been widely adopted in studies using the VHGT. Although the VHGT assesses all five levels, the analysis showed that the participants attained only Levels 1 (Visualization), 2 (Analysis), and 3 (Informal Deduction). No student attained Level 4 (Deduction), and consequently Level 5 (Rigor) was not attained because the Van Hiele model is hierarchical. Therefore, only Levels 1 - 3 are reported in the results.

3.7. Data Collection Procedure

Permission to conduct the study was obtained from the heads of the selected senior high schools in the Asante Akim South Municipality before data collection commenced. The Van Hiele Geometry Test (VHGT) developed by Usiskin [1] was administered to the participants in February 2024 under standardized examination conditions. Students were given 35 minutes to complete the 25-item multiple- choice test.

The VHGT consists of 25 items, with five items representing each Van Hiele geometric thinking level. Students' responses were scored according to the Van Hiele scoring procedure proposed by Usiskin [1]. A student was considered to have attained a particular Van Hiele level if he or she answered at least three of the five items correctly for that level. Based on this criterion, each student was assigned the highest Van Hiele geometric thinking level attained, thereby producing one geometric thinking level for each of the 320 students in the study.

The individual Van Hiele level attained by each student constituted the dataset entered into IBM SPSS Statistics version 26. Consequently, the descriptive statistics, including the mean (M = **1.875**) and standard deviation (SD = 0.741), were computed from the 320 individual student observations, where each student represented one case in the dataset. The frequencies and percentages reported in the results section were subsequently generated by summarizing the number of students who attained each Van Hiele level.

For example, if four students attained Level 2 and five students attained Level 1, SPSS first recorded each student as an individual observation (e.g., 2, 2, 2, 2, 1, 1, 1, 1, 1). The software then calculated the overall descriptive statistics from all student observations before producing the frequency distribution across the Van Hiele levels.

Therefore, the reported mean (**1.875**) and standard deviation (0.741) summarize the individual student-level data, whereas the frequency table is a descriptive summary showing the number of students who attained each Van Hiele level. The frequency table itself was not the dataset used to compute the SPSS descriptive statistics.

4. Results and Discussion

Table 2. Distribution of year three senior high school students across van Hiele geometric thinking levels (N = 320).

Van Hiele Level	Description	N	%
Level 1	Visualization	110	34.38
Level 2	Analysis	140	43.75
Level 3	Informal Deduction	70	21.88
Total		320	100.00

The distribution of Year Three Senior High School students across the Van Hiele geometric thinking levels is presented in **Table 2**. Of the 320 students, 110 (34.38%) attained Level 1 (Visualization), 140 (43.75%) attained Level 2 (Analysis), and 70 (21.88%) attained Level 3 (Informal Deduction). The findings indicate that Level 2 (Analysis) was the most frequently attained level, representing nearly half of the participants. The study adopted Usiskin's [1] numbering convention, in which the first Van Hiele level is designated as Level 1 rather than Level 0. Although the VHGT assesses five hierarchical levels, no student attained Level 4 (Deduction), and consequently no student attained Level 5 (Rigor). Therefore, only Levels 1 - 3 are reported.

Table 3. Descriptive statistics.

Statistic	Value
Mean	**1.875**
Standard deviation	0.741

As shown in **Table 3**, the mean Van Hiele geometric thinking level was **1.875** (SD=0.741), indicating that, on average, students operated at the Analysis level.

These findings suggest that the Analysis level (Level 2) represents the dominant stage of geometric thinking among the students. At this level, learners are able to identify and describe the properties of geometric figures but often experience difficulty constructing logical arguments and formal geometric proofs. Although 21.88% of the students attained the Informal Deduction level (Level 3), the relatively small proportion indicates that many learners have not yet developed the higher-order reasoning skills required for advanced geometric problem-solving. This finding underscores the need for instructional strategies that deliberately support students' progression to higher Van Hiele levels through carefully designed learning experiences.

The findings are consistent with those reported by Asemani et al. [7] and Burger and Shaughnessy [9], as well as the theoretical framework proposed by Van Hiele [3], which indicate that many secondary school students remain at the Analysis level, while relatively few attain higher levels of geometric reasoning. This consistency strengthens the evidence that geometry instruction should emphasize activities that promote logical reasoning, justification, and deductive thinking. Furthermore, this study contributes to mathematics education by providing recent evidence on the distribution of Van Hiele geometric thinking levels among Year Three Senior High School students in the Asante Akim South Municipality of Ghana. The findings provide valuable information for teachers, curriculum developers, and researchers seeking to improve geometry instruction and enhance students' geometric reasoning.

5. Conclusion

Based on the findings of this study, it is concluded that the largest proportion of Year Three Senior High School students in the Asante Akim South Municipality operate at the Analysis level (Level 2) of the Van Hiele geometric thinking model. Although some students have progressed to the Informal Deduction level (Level 3), relatively few have attained higher levels of geometric reasoning. These findings suggest that many students have developed the ability to identify and analyse the properties of geometric figures but require further instructional support to develop higher-order reasoning and deductive thinking. The study highlights the need for geometry instruction that deliberately promotes students' progression through the hierarchical levels of the Van Hiele model.

Recommendations

Based on the findings of this study, the following recommendations are made. First, mathematics educators and curriculum planners should design and implement geometry instruction that is explicitly aligned with the Van Hiele theory. Instruction should be sequenced to support students' progression through the hierarchical levels of geometric thinking, particularly from the.

Analysis level to the Informal Deduction level. Second, teachers should incorporate instructional activities that promote spatial reasoning, visualization, exploration, and logical reasoning. Learner-centred approaches, including the use of manipulatives, dynamic geometry software, and collaborative problem-solving activities, should be employed to enhance students' understanding of geometric concepts.

Third, school administrators should organize regular professional development programmes and workshops to strengthen teachers' knowledge of the Van Hiele theory and effective instructional strategies for promoting higher levels of geometric thinking. Finally, future studies should examine instructional interventions that facilitate students' progression from the Analysis level to the Informal Deduction and Deduction levels.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Usiskin, Z. (1982) Van Hiele Levels and Achievement in Secondary School Geometry (Final report of the Cognitive Development and Achievement in Secondary School Geometry Project). University of Chicago.

[2] Jablonski, S., & Ludwig, M. (2023). Teaching and learning of geometry—A literature review on current developments in theory and practice. *Education Sciences, 13*(7), 682. https://doi.org/10.3390/educsci13070682

[3] Van Hiele, P.M. (1986) Structure and Insight: A Theory of Mathematics Education. Academic Press.

[4] Vygotsky, L.S. (1978) Mind in Society: The Development of Higher Psychological Processes. Harvard University Press.

[5] Piaget, J. (1970) Science of Education and the Psychology of the Child. Orion Press.

[6] Usiskin, Z. (1987) Resolving the Continuing Dilemmas in School Geometry. In: Lindquist, M.M. and Schulte, A.P., *Learning and Teaching Geometry, K*-12: 1987 *Yearbook*, National Council of Teachers of Mathematics, 17-31.

[7] Asemani, E., Asiedu-Addo, S. K., & Oppong, R. A. (2017). The geometric thinking levels of senior high school students in Ghana. *International Journal of Mathematics and Statistics Studies, 5*(3), 1–8. https://www.eajournals.org/wp-content/uploads/The-Geometric-Thinking-Levels-of-Senior-High-School-Students-in-Ghana.pdf

[8] Cohen, L., Manion, L. and Morrison, K. (2007) Research Methods in Education. 6th Edition, Routledge. https://doi.org/10.4324/9780203029053.

[9] Burger, W.F. and Shaughnessy, J.M. (1986) Characterizing the van Hiele Levels of Development in Geometry. *Journal for Research in Mathematics Education*, **17**, 31-48. https://doi.org/10.2307/749317

Appendices

Appendix A

Section A

Adapted Van Hiele Geometry Test (Based on Usiskin, 1982)

This instrument was adapted from Usiskin [1] to assess students' levels of geometric thinking. It consists of 25 multiple-choice items distributed across the five Van Hiele levels: Visualization, Analysis, Informal Deduction, Deduction, and Rigor. Students select the one best answer for each item.

Level 1 - Visualization

1. Which of the following figures is a square?

(a) A shape with all sides equal and all right angles
(b) A shape with two long sides and two short sides
(c) A triangle with equal sides
(d) A shape with curved sides

2. Which figure below is not a triangle?

(a) A shape with three sides (b) A figure with four corners
(c) A figure with three corners (d) A closed figure made of three straight lines

3. A rectangle can best be recognized because:

(a) It looks like a box (b) It has four right angles
(c) It has all sides equal (d) It has only two right angles

4. Which figure is a circle?

(a) A figure with three equal sides (b) A figure with no corners
(c) A figure with four sides (d) A figure with two equal sides

5. Which of the following figures is not a polygon?

(a) Triangle (b) Rectangle (c) Circle (d) Pentagon

Level 2 - Analysis

6. Which statement about a parallelogram is true?

(a) It has one pair of parallel sides (b) Opposite sides are parallel and equal
(c) All angles are right angles (d) All sides are unequal

7. The diagonals of a rectangle:

(a) Are equal in length (b) Are always perpendicular
(c) Never meet (d) Are longer on one side

8. In a rhombus, the diagonals:

(a) Are equal and perpendicular (b) Are not equal but perpendicular
(c) Are not equal and not perpendicular (d) Are equal but not perpendicular

9. The sum of interior angles of a triangle is:

(a) 90° (b) 180° (c) 270° (d) 360°

10. A trapezium (trapezoid) has:

(a) All sides equal
(b) Two pairs of parallel sides
(c) One pair of parallel sides
(d) No parallel sides

Level 3 - Informal Deduction

11. Every square is a rectangle because:

(a) Both have four equal sides
(b) Both have all angles equal to 90°
(c) A square has all properties of a rectangle
(d) They look the same

12. Which statement is correct?

(a) Every rectangle is a rhombus
(b) Every square is a rhombus
(c) Every rhombus is a rectangle
(d) Every parallelogram is a rectangle

13. If a figure is a rectangle, then:

(a) Its opposite sides are equal and parallel
(b) It has only one right angle
(c) Its diagonals never meet
(d) Its diagonals are unequal

14. Which statement correctly relates triangles?

(a) All equilateral triangles are isosceles
(b) All isosceles triangles are equilateral
(c) Some scalene triangles are equilateral
(d) Right triangles are always equilateral

15. A kite is different from a parallelogram because:

(a) It has opposite sides parallel
(b) It has two pairs of adjacent equal sides
(c) It has all sides equal
(d) It has all right angles

Level 4 - Deduction

16. If two triangles have all three sides equal, then they are:

(a) Congruent (b) Similar only (c) Different in size (d) Right-angled

17. The sum of angles in a quadrilateral equal:

(a) 90° (b) 180° (c) 270° (d) 360°

18. If two lines are parallel and cut by a transversal, alternate interior angles are:

(a) Equal (b) Complementary (c) Supplementary (d) Unequal

19. The diagonals of a square are:

(a) Equal and perpendicular (b) Unequal and perpendicular
(c) Equal and not perpendicular (d) Unequal and not perpendicular

20. If the radius of a circle is doubled, its area:

(a) Doubles (b) Becomes four times larger
(c) Remains the same (d) Is halved

Level 5 - Rigor

21. In non-Euclidean geometry, the sum of interior angles of a triangle can be:

(a) Always 180° (b) Less or greater than 180°
(c) Exactly 360° (d) Cannot be measured

22. Two straight lines on a sphere (great circles):

(a) Never meet (b) Meet at two points
(c) Are parallel everywhere (d) Are perpendicular

23. In Euclidean geometry, through a point not on a line there is:

(a) One parallel line (b) No parallel line
(c) Many parallel lines (d) A perpendicular line only

24. A theorem in geometry is:

(a) A definition (b) A statement that can be proved
(c) A postulate (d) A guess

25. If two lines are perpendicular to the same line, they are:

(a) Parallel to each other (b) Perpendicular to each other
(c) Intersecting (d) Skew

Appendix B

Table B1 Marking scheme

Instructions to Examiner:

Use the answers to score each respondent's highest Van Hiele level (from Level 1 to Level 5).

Based on: Usiskin [1] - Cognitive Development and Achievement in Secondary School Geometry (CDASSG)

Level	Van Hiele Level	Question Nos.	Expected Student Performance	Mark Allocation (per question)	Level Mastery Criteria
L1	Recognition (Visualization)	1–5	Correctly identifies shapes (e.g., triangle, square, and circle) without necessarily knowing properties.	1 mark each (Total = 5 marks)	Achieves at least 3/5 correct responses.
L2	Analysis (Properties)	6–10	Recognizes and states correct properties of figures (e.g., "A rectangle has opposite sides equal").	1 mark each (Total = 5 marks)	Achieves at least 3/5 correct responses.
L3	Order / Informal Deduction (Abstraction)	11–15	Classifies shapes correctly (e.g., "All squares are rectangles but not all rectangles are squares").	1 mark each (Total = 5 marks)	Achieves at least 3/5 correct responses.
L4	Deduction (Formal Deduction)	16–20	Provides valid logical reasoning or proofs (e.g., applies Pythagoras theorem or angle sum theorem).	1 mark each (Total = 5 marks)	Achieves at least 3/5 correct responses.
L5	Rigor (Axiomatic Systems)	21–25	Demonstrates ability to compare Euclidean and non-Euclidean systems; uses axioms or postulates correctly.	1 mark each (Total = 5 marks)	Achieves at least 3/5 correct responses.